Final Examination: Dawson College: Department of Mathematics: Winter 2012
201-NYA-05: Calculus I (Regular - Social Science)

Question 1. If the limit exists, find its value; otherwise explain why the limit doesn't exist.

a. *(3 marks)*
$$\lim_{x \to 9} \frac{3}{x - 3}$$
| Answer: DNE |

b. *(3 marks)*
$$\lim_{x \to \infty} \frac{3x^3 - x + 1}{2 - 5x^3}$$
| Answer: $-\frac{3}{5}$ |

c. *(3 marks)*
$$\lim_{x \to 2} \frac{x^2 - 4}{x^4 + 2x^3}$$
| Answer: $\frac{1}{2}$ |

Question 2. *(5 marks)* For which values of x is the following function continuous? Clearly explain your reasoning.

$$f(x) = \begin{cases} \frac{3x-2}{x^2-1} & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ x^4 + x^3 + x + 2 & \text{if } x > 2 \end{cases}$$

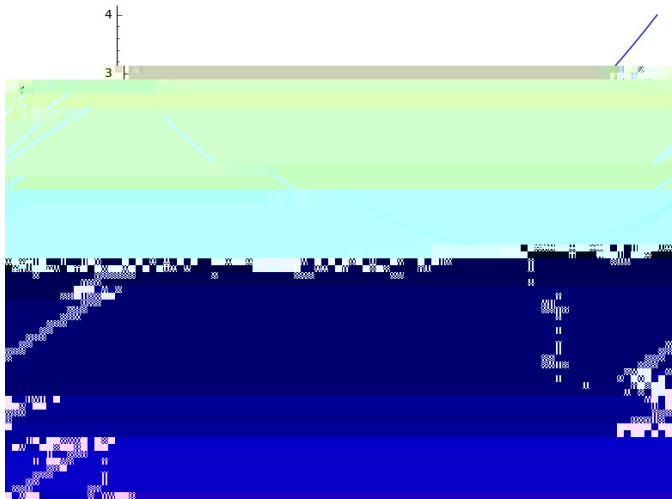| Answer: $\mathbb{R} nf \ 2g$ |

Question 3. Given
$$f(x) = x^2 - 2x + 1$$

a. *(4 marks)* Find the derivative of $f(x)$ using the definition of the derivative as a limit.
| Answer: $f'(x) = 2x - 2$ |

b. *(3 marks)* Find the tangent to $f(x)$ at $x = 2$.
| Answer: $y = 2x - 3$ |

c. *(3 marks)* Sketch the graph of $f(x)$ and its tangent at $x = 2$.

Question 4. (4 marks) Find the absolute maximum value and the absolute minimum value of the given function

$$f(x) = \frac{x}{x^2 + 1} \quad \text{on} \quad [-1;1]:$$

Answer: abs. min. $f(-1) = -\frac{1}{2}$, abs. max. $f(1) = \frac{1}{2}$

Question 5. Find the derivative of the following functions:

a. (5 marks)

$$f(x) = \arcsin e^{2x}$$

Answer: $f'(x) = \frac{2e^{2x}}{1 - e^{4x}}$

b. (5 marks)

$$f(x) = \frac{(\ln x + x^2)^2}{\sin x}$$

Answer: $f'(x) = \frac{(\ln x + x^2)\left[\left(\frac{2}{x} + 4x\right)\sin x - (\ln x + x^2)\cos x\right]}{\sin^2 x}$

c. (5 marks)

$$f(x) = (x^2 + 1)\arctan x$$

Answer: $f'(x) = 1 + 2x\arctan x$

Question 6. (5 marks) Using logarithmic differentiation, find the derivative of the function

$$f(x) = (x^3 + 1)^{12}\sqrt[3]{2x^2 + 5x}(x\tan x)^3:$$
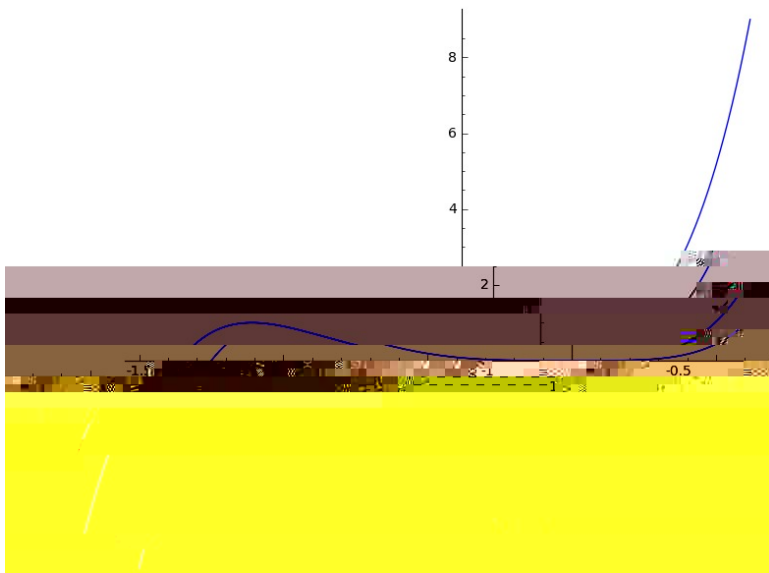
Do not simplify (expand) your answer.

Answer: $f'(x) = (x^3 + 1)^{12}\sqrt[3]{2x^2 + 5x}(x\tan x)^3\left[\frac{36x^2}{x^3 + 1} + \frac{4x + 5}{6x^2 + 15x} + \frac{3}{x} + \frac{2\sec^3 x}{\tan x}\right]$

Question 7. (5 marks) Find an equation of the tangent line to the curve $x^2y^3 - y^2 + xy - 1 = 0$ at the point $(1;1)$.

Answer: $y = -\frac{3}{2}x + \frac{5}{2}$

Question 8. (5 marks)

a. *(3 marks)* Show that the demand equation is $p = -0.0000002x$

Question 12. *(4 marks)* Find the horizontal asymptote(s) and vertical asymptote(s) (if any) of the following function.

$$f(x) = \frac{x^2 \quad 2x}{x^2 \quad 1}$$

Answer: $x = 1 ; x = \quad 1 ; y = 1$

Question 13. *(4 marks)* Evaluate the following integral.

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