Dawson College
Mathematics Department
Final Examination
201-BZF-05 Calculus III

Monday December 20th, 2010

Student Name: ______________________________

Student I.D. #: ______________________________

Instructor: Richard Fournier

TIME: 9:30 – 12:30 (3 [illegible]

INSTRUCTIONS:

- Print your name and student I.D. number in the space provided above.
- Answer 15 questions out of 20 (7 marks/question).
- All questions are to be answered directly on the examination paper.
- Translation and regular dictionaries are permitted.
- Small, noiseless, NON-PROGRAMMABLE calculators without text storage or graphic capabilities are permitted.
- Please ensure that you have a complete exam package before starting.

The exam must be returned intact.

Question #	*Marks*
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

1. [7 marks] Parametrize the curve with respect to arc length:

$$r(t)=\left(e^t\cos(2t),\ e^t\sin(2t),\ 2+\pi\right),\ 0\le t\le\pi.$$

$t = \ln\left(\frac{s}{\sqrt{5}}+1\right)$, $\quad 0 \le s \le \sqrt{5}(e^{\pi}-1)$

2. [7 marks] Find the angle between T and T' at $t=1$ if $r(t)=\left(t,t^2,t^3\right)$.

$0 = \frac{d}{dt}|T|^2 = \frac{d}{dt}\, T\cdot T = 2\, T\cdot T' \Rightarrow \angle(T, T') = 90^0$ or $\frac{\pi}{2}$ rad.

3. [7 marks] Compute $\lim \frac{xy\sin(x+y)}{\ldots}$.

0

4. [7 marks] Find the equation of the tangent plane to the surface $x^4+y^4+z^4=2$ at $(1,1,0)$.

$x+y=2$

7. [7 marks] Prove that $\left(\frac{x^2+y^2+z^2}{3}\right)^2 \le \frac{x^4+y^4+z^4}{3}$ for x, y, z real.

HINT: Maximize $x^2+y^2+z^2$ under the constraint $x^4+y^4+z^4=1$.

Call $f(x,y,z) = x^2+y^2+z^2$ and $g(x,y,z) = x^4+y^4+z^4-1$

According to Lagrange, $\exists L$: $\nabla f + L\nabla g = (0,0,0)$

Max $= \sqrt{3}$

8. [7 marks] Find all critical points of $f(x,y)=(1-x^2-y^2)^2$ and classify them.

at $(0,0)$: <u>local max</u>

at any (x,y) with $x^2+y^2=1$: <u>local min</u>

9. [7 marks] Assuming that the function $f(x,y)=\sqrt{xy}$ is differentiable at $(1,1)$, find an approximation to $\sqrt{(1.001)(1.0001)}$.

$\partial^2 f \quad \partial^2 f \quad \ldots \quad e^y + e^{-y} \quad \ldots \quad e^y - e^{-y}$

13. [7 marks] Compute $\left(\iint_R \left(x^2+y^2\right) dA\right)^2$ where $R=[0,1]x[0,1]$.

$\frac{4}{9}$

14. [7 marks] Compute $\iint_R e^{x^2+y^2} dA$ where $R=\left\{(x,y) \mid 0 \le x^2+y^2 \le 1\right\}$.

$\pi(e-1)$

15. [7 marks] Compute

$\iiint e^{x^2+y^2} dV$ where R is the cylinder $\left\{(x,y,z) \mid x^2+y^2 \le 1 \text{ and } 0 \le z \le 1\right\}$.

16. [7 marks] Compute

$\iiint_R dV$ where R is the sphere $\left\{(x,y,z) \mid x^2+(y-1)^2+(z-2)^2 \le 2\right\}$.

$\frac{8\sqrt{2}\pi}{3}$

17. [7 marks] Find a power series representation

18. [7 marks] Test for convergence: $\sum_{n=1}^{\infty}(-1)^{n-1}\left(n^{1/n}-1\right)$.

$n^{1/n}$ decreases to 1 as n increases!

the series converges (Leibniz!)

19. [7 marks] Find the interval of convergence for $\sum_{n=2}^{\infty}\frac{2^n}{n^2}(x-2)^{4n}$.

$\left[2-\frac{1}{\sqrt[4]{2}},\ 2+\frac{1}{\sqrt[4]{2}}\right]$

20. [7 marks] Find the sum of the series $\sum_{n=1}^{\infty}nx^{2n}$. $\frac{x^2}{(1-x^2)^2}$, $|x|<1$