DAWSON COLLEGE

MATHEMATICS DEPARTMENT

Final Examination

Mathematics 201-NYC-05 | Date: Friday, December 14, 2012

Linear Algebra (Commerce) | Time: 2:00 - 5:00

Instructors: Rodney Acteson, Melanie Beck, Olga Zlotchevskaia

1. **(6 marks)** Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{array}{rcrcrcr} & & 2y & & 5z & = & 8 \\ 2x & & 6y & + & 4z & = & 8 \\ 3x & + & 5y & & 16z & = & 4 \end{array}$$

2. **(6 marks)** Consider the system

$$\begin{array}{rcrcrcr} x & + & 3y & + & 5z & = & 2 \\ x & & 2y & & 3z & = & k \\ 3x & & 8y & & 13z & = & 5 \end{array}$$

For which value(s) of k is the system

(a) consistent?

(b) inconsistent?

3. **(3 marks)** Determine whether the following statement is true or false. If the answer is True, justify your answer, if the answer is False, provide an example which shows that the statement is False.

$$(A + B)(A \quad B) = A^2 \quad B^2; \qquad \text{for all square matrices } A \text{ and } B \text{ of the same size.}$$

4. **(5 marks)** Simplify $(AB)^{\;1}(AC)(A^TC)^{\;1}$.

5. **(6 marks)** Find A such that

$$(A^T \quad 8I)^{\;1} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

6. **(4 marks)** Find elementary matrices E_1 and E_2 which satisfy the following equation.

$$E_2E_1\begin{bmatrix} 5 & 6 \\ 0 & 1 \end{bmatrix} = I$$

7. **(8 marks)** Given $A = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$, nd

(a) $\det(A)$

(b) $\operatorname{adj}(A)$

(c) $A^{\;1}$ using $\operatorname{adj}(A)$

(d) The solution to $AX = B$ using $A^{\;1}$

8. **(5 marks) Using only row operations,** nd the determinant of the following matrix A .

$$A = \begin{bmatrix} 2 & 0 & 3 & 2 \\ 3 & 1 & 3 & 7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 9 \end{bmatrix}$$

9. **(9 marks)** Let A and B be 3×3 matrices with $\det(AB) = 24$ and $\det(A) = -6$. Find

   (a) $\det(-3A^{-1})$
   (b) $\det(BAB)$
   (c) $\det(A^T \det(A))$

10. **(6 marks)** Consider the vectors $u = (2, -2, 3)$ and $v = (2, 2, 1)$.

    (a) Find a unit vector perpendicular to both u and v.
    (b) Find the area of the parallelogram defined by the vectors u and v.

11. **(4 marks)** Consider the vectors $u = (2k, -1, -k)$ and $v = (4, 2, -2)$.

    (a) For which value(s) of k, are the vectors u and v perpendicular?
    (b) For which value(s) of k, are the vectors u and v parallel?

12. **(4 marks)** Find the equation of the line passing through $P(3, -1, 2)$ and perpendicular to the plane $4x - 5y = 2z + 3$.

13. **(6 marks)** Given the two planes

$$2(x - 1) + (y - 1) + 3(z + 4) = 0 \qquad \text{and} \qquad x + y - 4z = 2,$$

    (a) Show that the planes are not parallel.
    (b) Find the line of intersection of the planes.

14. **(5 marks)** Find the point on the line

$$x = -1 + 3t,\ y = 2 - 2t,\ z = 1 - t,\ (-\infty < t < \infty),$$

that is closest to $P(-1, 5, 9)$.

15. **(6 marks)** Find an equation for the plane that contains the point $P(-2, 1, 3)$ and the line

$$x = 1 + 2t,\ y = -2 + t,\ z = 2 - t,\ (-\infty < t < \infty).$$

16. **(5 marks)** Prove the following identity

$$(u + kv) \times v = u \times v,$$

where k is a scalar and u and v are vectors in 3-space.

17. **(4 marks)** A manufacture is producing bikes and scooters. It takes 6 hours to make a bike and 4 hours to make a scooter. Each bike requires 7 kg of steel and each scooter requires 5 kg of steel. The manufacture has 40 hours available for making bikes and scooters and has 280 kg of steel on hand. The company makes a profit of $25 on each bike and $15 on each scooter. How many bikes and how many scooters should it make in order to maximize he profit?
Set up the linear programming problem as follows **(you are not asked to solve the problem!)**:

    (a) Define all the variables (using the phrase "the number of").
    (b) State the objective and identify the objective function (in terms of the variables).
    (c) State all the constraints (in terms of the variables).

18. **(8 marks)** Solve the following problem.

$$\begin{array}{lrl} \textbf{Maximize} & p = 4x + 2y - 2z & \\ \textbf{subject to} & x - y - z & \le 20 \\ & 2x + y + 2z & \le 70 \\ & x + z & \le 10 \\ & x, y, z & \ge 0 \end{array}$$

Answers:

1. $x = 8 \quad \frac{19}{2}t;\ y = 4 \quad \frac{5}{2}t;\ z = t$

2. (a) if $k = 1$ (b) if $k \neq 1$

3. False: example $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

4. $B^{\ 1}(A^T)^{\ 1}$

5. $A = \begin{matrix} 21{=}2 & 2 \\ 3{=}2 & 7 \end{matrix}$

6. $E_1 = \begin{matrix} 1{=}5 & 0 \\ 0 & 1 \end{matrix}$ $E_2 = \begin{matrix} 1 & 6{=}5 \\ 0 & 1 \end{matrix}$