

Exam

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THIS

- (1) Find a power series representation, and its interval of convergence.

(Hint: $x^2 - 2x = (x-1)^2 - 1$.)

$$\frac{1}{x^2 - 2x} = \frac{1}{-1 + (x-1)^2} = \frac{-1}{1 - (x-1)^2}$$

The series converges only if $|x-1| < 1$ and the interval of convergence is $(0, 2)$.

- (2) Find the exact value of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$. (Hint: $\frac{d}{dx} \frac{x}{(1-x)^2} = \frac{1+x}{(1-x)^3}$)

$$\frac{d}{dx} \frac{x}{(1-x)^2} = \frac{1+x}{(1-x)^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$$

$$\frac{1-x}{(1+x)^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$$

$$x \frac{1-x}{(1+x)^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$$

$$\sqrt{2} (\sqrt{2}-1)^4 = \frac{1}{\sqrt{2}} \frac{1-1/\sqrt{2}}{(1+1/\sqrt{2})^3} = \frac{1}{\sqrt{2}} \frac{1-\sqrt{2}}{(1+\sqrt{2})^3}$$

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(4) Find

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(5) Find

(6) Comp

$$\left\langle \frac{1-}{1+} \right\rangle$$

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(9) Compute the limit

We have

so that

$$\lim_{(x,y) \rightarrow (0,0)}$$

(10) Show that all tange.

At a point

the gradient

and the

$$(-2^{y_0})$$

Since

$$-x_0$$

$$= -x_0$$

$$= 0,$$

any

(11)

(12)

8 7

(13) Find

at

at

(14) Find

(15) Find the (absolute) minimum of the function $f(x, y)$:

Over the disc, we have

$$0 \leq x^2 + y^2$$

$$e^0 \leq e^{x^2 + y^2}$$

$$e^{-2} = \frac{1}{e^2} \leq e^{-x^2 - y^2}$$



the min value
attained at any
point of the
circle with equation $x^2 + y^2 = 2$

(16) Compute the integral $\iint_R (x-y)^2 dA$ over the rectangle

By Fubini, this is

$$\int_{-1}^1 \left(\int_{-2}^2 (x-y)^2 dy \right) dx$$

$$\int_{-1}^1 \left[-\frac{(x-y)^3}{3} \right]_{-2}^2 dx$$

$$\int_{-1}^1 \left(-\frac{(x-2)^3}{3} + \right.$$

$$\left. -\frac{(x-2)^4}{12} + \right) dx$$

$$-\frac{1}{12} + \frac{3^4}{12} + \frac{3^4}{12}$$

(17) Compute the integ

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \arcsin x \Big|_0^1 = \frac{\pi}{2}$$

(18) Compute the volun

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2} dy dx = \int_0^1 \left[-\frac{1}{2} \sqrt{1-x^2-y^2} + \frac{1}{2} \arcsin \frac{y}{\sqrt{1-x^2}} \right]_0^{\sqrt{1-x^2}} dx$$

$$= \int_0^1 \left[-\frac{1}{2} \sqrt{1-x^2} + \frac{1}{2} \arcsin 1 \right] dx = \int_0^1 \left[-\frac{1}{2} \sqrt{1-x^2} + \frac{\pi}{4} \right] dx$$

$$= \left[-\frac{1}{2} \left(x \sqrt{1-x^2} + \arcsin x \right) + \frac{\pi}{4} x \right]_0^1 = -\frac{1}{2} \left(\frac{1}{2} + \frac{\pi}{2} \right) + \frac{\pi}{4} = \frac{\pi}{4} - \frac{1}{4}$$

(19) Use a double integral to find the area of the circle $x^2 + y^2 = 4$.

the circle is
the "maxim

By using
the region
 \iint_D
 \ominus

(20) Compute the volume of a sphere of radius 1. (Hint: use spherical coordinates, which $dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$.)

The integral
 $\int_0^\pi \int_0^{2\pi} \int_0^1$

$$= 2\pi$$

$$= 2\pi$$

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