

Dawson College Department of Mathematics – Winter 2011

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**Final Examination Tuesday May 17th 2011
Calculus I (201-103-DW)**

STUDENT NAME: _____

**No graphing / programmable calculators allowed.
There are 16 questions in total worth 100 marks.
Show all your work in the space provided and circle your final answers.**

$$(a) \lim_{x \rightarrow 2} \frac{3x^2 + 6x - 24}{x^2 - 4} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 2} \frac{3(x-2)(x+4)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{3(x+4)}{x+2} = \left(\frac{9}{2} \right)$$

$$(b) \lim_{x \rightarrow 1} \frac{x}{\sqrt{1-3x} - 1} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{x}{\sqrt{1-3x} - 1} \cdot \frac{(\sqrt{1-3x} + 1)}{(\sqrt{1-3x} + 1)}$$

[3] 2. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt[3]{2+x-3x^2} & \text{if } x \geq 2 \\ 3x-8 & \text{if } x < 2 \end{cases}$$

Find the limits:

$$(a) \lim_{x \rightarrow 2^+} f(x) = \sqrt[3]{2+2-12} = -2$$

$$(b) \lim_{x \rightarrow 2^-} f(x) = 6-8 = -2$$

$$x^2 - x - 12$$

is not 2

definition to evaluate $f'(x)$ for $f(x) = \frac{3}{2}x^2 + 7x - 1$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{3}{2}(x+h)^2 + 7(x+h) - 1 - (\frac{3}{2}x^2 + 7x - 1)}{h}$$

16. 5 Find the equation of the tangent line to the graph of $f(x) = \frac{x}{7-3x}$

at the point where $x = 1$.

$$m = f'(1)$$

$$f'(x) = \frac{(7-3x)^{1/2} (1) - x \frac{1}{2} (7-3x)^{-1/2} (-3)}{(7-3x)}$$

$$= f'(1) = \frac{11}{16} \rightarrow y = \frac{11}{16}x + b$$

$$= f(1) = \frac{1}{2}$$

$$\frac{1}{2} = \frac{11}{16} + b$$

$$b = -\frac{3}{16}$$

$$y = \frac{11}{16}x - \frac{3}{16}$$

[5] 6. For the function $f(x) = \frac{2x+1}{x-1}$ find $f''(x)$ and simplify your answer.

$$= -3(x-1)^{-2}$$

$$f''(x) = 6(x-1)^{-3}$$

$$= \frac{6}{(x-1)^3}$$

[5] 7. Compute the value of $f'(x)$ when $x=0$ given that

$$f(x) = (e^{-2x} + 3)^3 + \sin(6x).$$

$$f'(x) = 3(e^{-2x} + 3)^2(e^{-2x})(-2) + 6\cos(6x)$$

[7] 8. Suppose the monthly revenue and cost functions (in dollars) for x units of a

commodity sold and produced are $R(x) = 400x - \frac{x^2}{20}$ and

$$C(x) = 5000 + 70x.$$

a) Find the Profit function $P(x)$.

$$P(x) = R(x) - C(x)$$

$$= -\frac{x^2}{20} + 330x - 5000$$

b) Find the marginal profit function and use it to estimate the profit from selling the 42nd unit.

$$P'(x) = -\frac{2x}{20} + 330 = -\frac{x}{10} + 330$$

$$P'(41) = -\frac{41}{10} + 330 = 325.90 \text{ \$/unit}$$

c) Calculate the actual profit from selling the 42nd unit.

$$P(42) - P(41) = 325.85 \text{ \$}$$

15] 9, Find the x-value(s) where the graph of $f(x) = \frac{3x-2}{\sqrt{2x+1}}$ has a horizontal

$$f'(x) = \frac{(2x+1)^{1/2} \cdot 3 - (3x-2) \cdot \frac{1}{2}(2x+1)^{-1/2} \cdot 2}{(2x+1)}$$

$$= \frac{(2x+1)^{-1/2} \left[(2x+1) \cdot 3 - (3x-2) \right]}{(2x+1)}$$

$$= \frac{6x+3 - 3x+2}{(2x+1)^{3/2}}$$

$$= \frac{3x+5}{(2x+1)^{3/2}} = 0$$

[6] 10. Consider the relation $\sqrt{x+y} = 1+x^2y^2$.

(a) Find the derivative y' .

$$\frac{1}{2} (x+y)^{-1/2} (1+y') = x^2 2y y' + 2xy^2$$

$$(x+y)^{-1/2} + (x+y)^{-1/2} y' = 4x^2 y y' + 4xy^2$$

$-1/2$

$$y' [(x+y)^{-1/2} - 4x^2 y] = -(x+y)^{-1/2} + 4xy^2$$

$$y' = - (x+y)^{-1/2} + 4xy^2$$

$$f(x) = (\cos 2x)^{\ln x}$$

$$y = (\cos 2x)^{\ln x}$$

$$\ln y = \ln (\cos 2x)^{\ln x}$$

$$\ln y = (\ln x) \cdot \ln (\cos 2x)$$

$$\frac{y'}{y} = \ln x \frac{(-\sin 2x) \cdot 2}{\cos 2x} + \ln (\cos 2x) \cdot \frac{1}{x}$$

[5] 12. Consider the function $f(x) = \ln\left(\frac{e^x \sqrt{x^2+1}}{(x-2)^3}\right)$.

(a) Fully simplify $f(x)$ using the properties of logarithms.

$$f(x) = \ln(e^x) + \frac{1}{2} \ln(x^2+1) - 3 \ln(x-2)$$

$$= x + \frac{1}{2} \ln(x^2+1) - 3 \ln(x-2)$$

(b) Find $f'(x)$ and simplify your answer.

$$f'(x) = 1 + \frac{1}{2} \frac{2x}{x^2+1} - 3 \frac{(1)}{x-2}$$

$$= 1 + \frac{x}{x^2+1} - \frac{3}{x-2}$$

[12] 13. Consider the function $f(x) = x^3 - 6x$

(a) Find the x and y intercepts.

y-int: $y = 0$

x-int: $x(x^2 - 6) = 0$
 \downarrow \downarrow
 $x = 0$ $x = \pm\sqrt{6}$
 $= \pm 2.45$

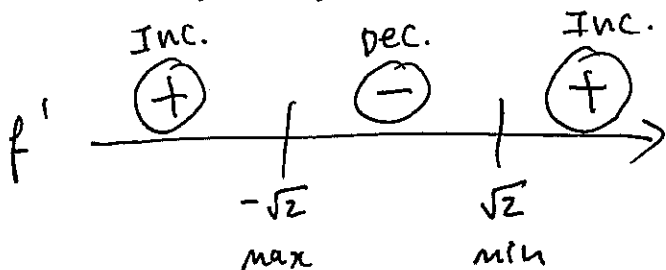
$(0, 0)$
 $(2.45, 0)$
 $(-2.45, 0)$

(b) Find the intervals of increase/decrease and the relative extrema.

$f'(x) = 3x^2 - 6 = 0$

$3(x^2 - 2) = 0$

\downarrow
 $x = \pm\sqrt{2} = \pm 1.41$

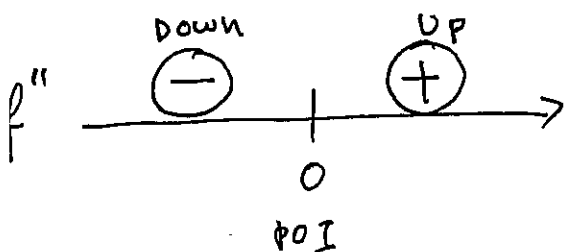


local max $(-1.41, 5.66)$
local min $(1.41, -5.66)$

(c) Find the intervals of concavity and the points of inflection.

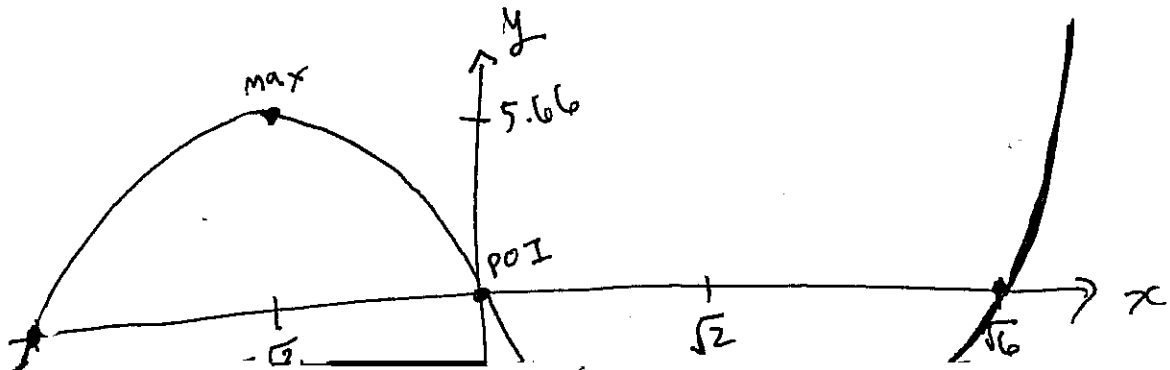
$f''(x) = 6x = 0$

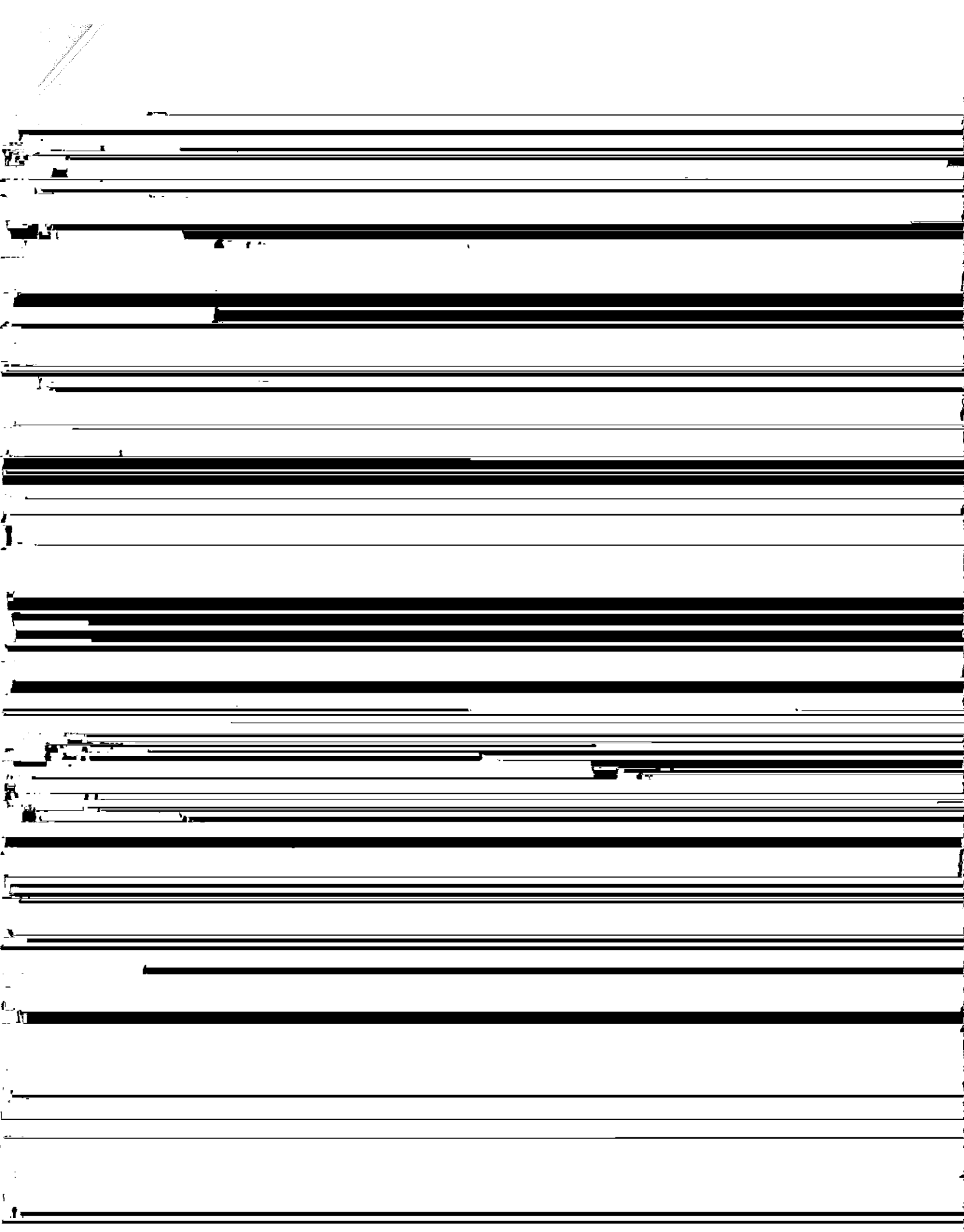
\downarrow
 $x = 0$



POI $(0, 0)$

(d) Sketch a graph of $f(x)$ with important points labeled.





[8] 16. Find the antiderivatives:

(a) $\int \sqrt{x}(2x^3 - 7) dx$

$\int (2x^{7/2} - 7x^{1/2}) dx$

$= \frac{4}{9} x^{9/2} - \frac{14}{3} x^{3/2} + C$

(b) $\int \frac{5x-1}{\sqrt[3]{5x^2-2x+1}} dx$

$\int \frac{du}{2 u^{1/3}}$

$u = 5x^2 - 2x + 1$
 $du = 10x - 2 dx$

$du = 2(5x-1) dx$

$\frac{du}{2} = (5x-1) dx$