

Dawson College Mathematics Department Winter 2010

Instructors: I. Rajput, K. Ameer, G. Honnouvo, M. Ishii

**Final Examination May 20th 2010
Calculus I (201-103-DW)**

STUDENT NAME: _____

U: _____

**No graphing/programmable calculators allowed.
There are 14 questions in total worth 100 marks.
Show all your work in the space provided and circle your final answers.**

[9] 1. Find each of the following limits:

$$(a) \lim_{x \rightarrow 5} \frac{x^2 + 2x - 35}{x^2 - 6x + 5}$$

$$= \lim_{x \rightarrow 5} \frac{(x+7)(\cancel{x-5})}{(x-1)(\cancel{x-5})}$$

$$= \frac{12}{4} = \textcircled{3}$$

$$(b) \lim_{x \rightarrow 49} \frac{x-49}{\sqrt{x}-7}$$

$$= \lim_{x \rightarrow 49} \frac{(\sqrt{x}-7)(\sqrt{x}+7)}{(\sqrt{x}-7)}$$

$$= \textcircled{14}$$

$$(c) \lim_{x \rightarrow \infty} \frac{1+x^2-3x^3}{2x^3+1}$$

same degree

$$\lim_{x \rightarrow \infty} \frac{-3x^3}{2x^3}$$

$$= \textcircled{-\frac{3}{2}}$$

[7] 2. Consider the piece-wise defined function:

$$f(x) = \begin{cases} \frac{-x^2}{2} - 2x & x \leq -1 \\ \frac{x}{2} + 2 & x > -1 \end{cases}$$

(a) Find $f(-1)$.

$$\begin{aligned} f(-1) &= \frac{-(-1)^2}{2} - 2(-1) \\ &= -\frac{1}{2} + 2 = \frac{3}{2} \end{aligned}$$

(b) Find $\lim_{x \rightarrow -1} f(x)$.

$$\begin{aligned} \lim_{x \rightarrow -1^+} f(x) &= \frac{-1}{2} + 2 = \frac{3}{2} \\ \lim_{x \rightarrow -1^-} f(x) &= \frac{3}{2} \end{aligned}$$

$$\lim_{x \rightarrow -1} f(x) = \frac{3}{2}$$

- [7] 3. State the limit definition of the derivative (4-step process) and use ONLY the limit definition to evaluate $f'(x)$ for $f(x) = -5x^2 - 4x + 3$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-5(x+h)^2 - 4(x+h) + 3 - (-5x^2 - 4x + 3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-5x^2 - 10xh - 5h^2 - 4x - 4h + 3 + 5x^2 + 4x - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-10xh - 4h - 5h^2}{h}$$

$$= \lim_{h \rightarrow 0} -10x - 4 - 5h = -10x - 4$$

- [4] 4. Consider the function $f(x) = \frac{-5}{3}x^2 + 2x + 2$. Find the equation of the tangent line to the graph of this function at $x = -1$.

$$f'(x) = -\frac{10}{3}x + 2$$

$$m = f'(-1) = -\frac{10}{3}(-1) + 2$$

$$= \frac{10}{3} + \frac{6}{3}$$

$$= \frac{16}{3}$$

$$y = f(-1) = \frac{-5}{3}(-1)^2 + 2(-1) + 2$$

$$y = mx + b$$

$$-\frac{5}{3} = \frac{16}{3}(-1) + b$$

$$-\frac{5}{3} + \frac{16}{3} = b$$

$$\frac{11}{3} = b$$

$$y = \frac{16}{3}x + \frac{11}{3}$$

[16] 5. Find the derivative for each function. (You do not have to simplify.)

$$(a) f(x) = \left(\frac{2x-1}{x^2+x} \right)^4$$

$$f'(x) = 4 \left(\frac{2x-1}{x^2+x} \right)^3 \left[\frac{(x^2+x)(2) - (2x-1)(2x+1)}{(x^2+x)^2} \right]$$

$$(b) f(x) = \sin(x^3) - \sqrt{\cos(2x)} + e^{\tan x}$$

$$\begin{aligned} f'(x) &= \cos(x^3) \cdot 3x^2 + \frac{1}{2} \frac{1}{\sqrt{\cos(2x)}} \sin(2x) \cdot 2 + e^{\tan x} \cdot \sec^2 x \\ &= 3x^2 \cos(x^3) + \frac{\sin(2x)}{\sqrt{\cos(2x)}} + e^{\tan x} \sec^2 x \end{aligned}$$

$$f(x) = \ln(x^2+3)$$

$$f(x) = \ln(x^2+2) - \ln(1-x)$$

$$f'(x) = \frac{2x}{x^2+3} + \frac{1}{|2(1-x)|}$$

$$(d) f(x) = x^4 e^{2x}$$

$$\begin{aligned} f'(x) &= x^4 e^{2x} \cdot 2 + e^{2x} 4x^3 \\ &= 2x^4 e^{2x} + 4x^3 e^{2x} \end{aligned}$$

[5] 6. Find $\frac{dy}{dx}$ given $y = x^{\sqrt{x}}$.

$$\ln y = \sqrt{x} \ln x$$

$$\frac{1}{y} y' = \sqrt{x} \frac{1}{x} + \ln x \cdot \frac{1}{2} x^{-1/2}$$

$$\frac{1}{y} y' = \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}$$

$$y' = x^{\sqrt{x}} \left(\frac{2 + \ln x}{2\sqrt{x}} \right)$$

[6] 7. Find $f''(2)$ if $f(x) = \sqrt{x^3 - 4}$. (Find the value of the second derivative when $x=2$.)

$$f'(x) = \frac{1}{2} (x^3 - 4)^{-1/2} \cdot 3x^2$$

$$= \left(\frac{3}{2} x^2 \right) (x^3 - 4)^{-1/2}$$

(a) Find the marginal cost function and use it to estimate the cost of producing the 11-th unit.

$$C'(x) = 3x^2 - 18x + 33$$

$$C'(10) = 3(100) - 18(10) + 33$$

$$= 153 \$$$

[6] 9. Consider the relation: $x^3 + y^3 = 6xy$.

(a) Find the derivative $\frac{dy}{dx}$.

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + 6y$$

$$3x^2 - 6xy = \frac{dy}{dx} (6x - 3y^2)$$

$$\frac{3x^2 - 6y}{6x - 3y^2} = \frac{dy}{dx}$$

$$\hookrightarrow \boxed{\frac{dy}{dx} = \frac{x^2 - 2y}{2x - y^2}}$$

(b) Find the equation of the tangent line to the curve of this relation at the

point (3, 3).

$$y = mx + b$$

$$m = \frac{(3)^2 - 2(3)}{2(3) - (3)^2}$$

$$= \frac{3}{3 - 9} = -\frac{1}{2}$$

$$\boxed{y = -x + 6}$$

- [5] 10. Consider the function $f(x) = (x^2 - 1)^3$. Find the absolute maximum and minimum values for $f(x)$ on the closed interval $[-1, 2]$.

$$f'(x) = 3(x^2 - 1)^2 \cdot 2x$$

$$0 = 6x(x^2 - 1)^2$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=0 & x=\pm 1 \end{array}$$

$$f(-1) = ((-1)^2 - 1)^3 = 0$$

$$f(0) = (-1)^3 = -1$$

absolute
min

$$f(1) = 0$$

$$f(2) = 27$$

absolute
max.

- [4] 11. Find the x-values where the graph of the function $f(x) = \frac{(x-1)^2}{(2x+1)^3}$ has a horizontal tangent line.

$$f'(x) = \frac{(2x+1)^3 \cdot 2(x-1) - (x-1)^2 \cdot 3(2x+1)^2 \cdot 2}{(2x+1)^6}$$

$$= \frac{2(x-1)(2x+1)^2 [(2x+1) - 3(x-1)]}{(2x+1)^6}$$

$$(2x+1)^6$$

$$0 = \frac{-2(x-1)(x-4)}{(2x+1)^4}$$

$$(2x+1)^4$$

[12] 12. Consider the function $f(x) = x^4 - 4x^3$.

(a) Find the x and y intercepts.

$$(0,0)$$

$$0 = x^3(x-4)$$

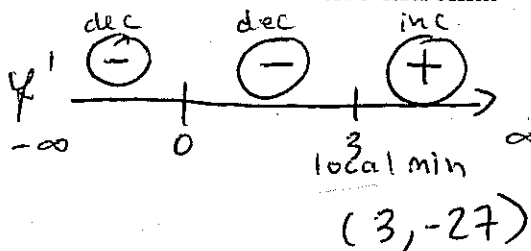
$$(0,0), (4,0)$$

(b) Find the intervals of increase/decrease and the relative extrema.

$$f'(x) = 4x^3 - 12x^2$$

$$0 = 4x^2(x-3)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=0 & x=3 \end{array}$$



(c) Find the intervals of concavity and the points of inflection.

$$f''(x) = 12x^2 - 24x$$

UP

DOWN

UP

- [7] 13. A manufacturer estimates that x units of its product can be produced at a total cost of $C(x) = x^3 + 100x + 45000$ dollars. If the total revenue from the sale of x units is $R(x) = 4600x$ dollars, determine the level of production

that will maximize the profit. Also, calculate the maximum profit.

$$P(x) = R(x) - C(x)$$

$$= 4600x - x^3 - 100x - 45'000$$

$$= -x^3 + 4500x - 45000$$

$$P'(x) = -3x^2 + 4500$$

$$0 = -x^2 + 1500$$

$$x^2 = 1500$$

$$x = \oplus 38.73 \approx 39 \text{ units.}$$

$$P'(2) = \oplus$$

$$P'(40) = \ominus$$

max at $x = 39$

$$P(39) = -59219 + 175500 - 45000$$

[6] 14. Find the antiderivatives:

$$(a) \int \frac{(x^4 + 3\sqrt{x})}{x^2} dx$$

$$= \int \frac{x^4}{x^2} dx + 3 \int \frac{x^{1/2}}{x^2} dx$$

$$= \int x^2 dx + 3 \int x^{-3/2} dx$$

$$= \frac{x^3}{3} + 3 x^{-1/2} (-2) + C$$

$$= \frac{x^3}{3} - \frac{6}{\sqrt{x}} + C$$

$$(b) \int x^2 \sqrt{x^3 + 1} dx$$

$$\left. \begin{aligned} u &= x^3 + 1 \\ du &= 3x^2 dx \end{aligned} \right\}$$

$$= \frac{1}{3} \int u^{1/2} du$$

$$= \frac{1}{3} u^{3/2} \cdot \frac{2}{3} + C$$

$$= \frac{2}{9} (x^3 + 1)^{3/2} + C$$