

$$(6) + (7) + (8)$$

$$y'' + y' + y = e^{2x} \{(-5 + 4 + 1) \sin 3x + (12 + 6) \cos 3x\} = 18 \cos 3x$$

Assignment for functions satisfying D.E.

- (1) $y = 2x^3 - x^2$; $xy' - 3y = x^2$.
- (2) $y = \frac{1}{2} \cos x$; $y'' + 9y = 4 \cos x$.
- (3) $y = \sin x + \cos x - e^{-x}$; $y' + y = 2 \cos x$.
- (4) $y = 3e^{-2x} + 4e^{-x}$; $y'' + 3y' + 2y = 0$.
- (5) $y = t \sin 2t$; $y'' + 4y = 4 \cos 2t$.
- (6) $y = 5xe^{-4x}$; $y'' + 8y' + 16y = 0$.
- (7) $y = e^{-3x} \sin 2x$; $y'' - 5y' = -12e^{-3x} \cos 2x$.

Applications of differential equations

- (1) When a hockey player strikes a puck with a certain force at $t = 0$ the puck moves along the ice with velocity at any time thereafter given by the expression $v(t) = 27 - 9 \sqrt{t}$ m/sec. How far would the puck travel on a long sheet of ice before coming to rest? (Assume $s(t) = 0$ at $t = 0$.)

Solution: $\frac{ds}{dt} = v(t)$. Therefore

$$\begin{aligned} s &= \int ds = \int v(t) dt = \int (27 - 9 \sqrt{t}) dt = 27t - \frac{9t^{3/2}}{3/2} + C \\ &= 27t - 6t^{3/2} + C. \end{aligned}$$

At $t = 0$, $s(t) = 0 = 27(0) - 6(0) + C$. Therefore $C = 0$. When the puck stops, $v(t) = 0 = 27 - 9 \sqrt{t}$. Hence $9 \sqrt{t} = 27$ so $\sqrt{t} = 3$. Therefore $t = 9$ sec. Substituting in the equation for s , $s(9) = 27(9) - 6(9)^{3/2}$

Solution: (a) Separating the variables we have

$$\frac{dv}{-v^2} = 0.04 dt \quad \text{or} \quad -v^{-2}dv = 0.04dt.$$

$\frac{-v^{-1}}{-1} = 0.04t + C$ or simplifying $\frac{1}{v} = 0.04t + C$. Now applying the initial conditions, when $t = 0$, $v = 10$ m/sec. so $1/10 = C$. Therefore

$$\frac{1}{v} = 0.04t + \frac{1}{10} = \frac{0.4t + 1}{10}. \quad \text{Inverting the equation } v = \frac{10}{1 + 0.4t}.$$

(b) When $t = 10$ sec.,

$$v = \frac{10}{1 + (0.4)(10)} = 10$$

- (4) a train is travelling at 64 km/hr . when the caboose suddenly becomes detached. If the friction of the rails provides a deceleration of $a = \frac{dv}{dt} = -2v^{3/2} \text{ km/hr}^2$, how fast will the caboose be going after $\frac{1}{4}$ of an hour?
(Hint: Solve the D.E. $-\frac{dv}{v^{3/2}} = 2dt$)
- (5) A population of whales grows at the rate $\frac{dP}{dt} = \frac{2}{3}P^{1/4}$. If $P = 16$ when $t = 0$ years, how many whales will there be after 38 years?
- (6) The volume of water in a tank is given by $V = 8h^{3/2}$. Therefore

$$h^{3/2} = \frac{V}{8} \quad \text{or} \quad h = \frac{V}{8}^{2/3} = \frac{V^{2/3}}{4}.$$

If the water drains from the tank according to Toricelli's law, then

$$\frac{dV}{dt} = -2\bar{h} = -2 \frac{\sqrt{V^{2/3}}}{4} = -2 \frac{V^{1/3}}{2} = -V^{1/3} \quad \frac{dV}{V^{1/3}} = -dt.$$

- (a) If the tank is initially filled with 125 m^3