DAWSON COLLEGE

MATHEMATICS DEPARTMENT

Final Examination

Mathematics 201-401-DW
Statistics for Social Science
(Section 01)
Instructor: Melanie Beck

Date: Tuesday, December 19, 2017
Time: 9:30 - 12:30

1. ***(6 marks)*** Find the probability that when two cards are drawn from a standard deck of 52 without replacement, the second card drawn is a heart. (You can use a tree diagram if you want to.)

2. ***(3 marks)*** The organizer of a contest must form a committee consisting of six persons. He has been suggested 5 names of artists, 3 of politicians and 6 of professors. How many di erent committees can be formed?

3. ***(8 marks)***

7. ***(6 marks)*** Let X be a continuous random variable that is normally distributed with a mean of 40 and a standard deviation of 4. Find the probability that X

of 23.4. Do the data suggest that there is a di erence in daily sales before and after the script change? Use = 0:05.

16. ***(7 marks)*** The makers of ink cartridges for color ink-jet printers have developed a new system for storing the ink. They think the new system will result in a longer lasting product. In order to determine whether this is the case, a test was developed in which a sample of 35 of the new cartridges was selected. They were put in a printer, and test pages were run until the cartridge was empty. The same thing was done for a sample of 32 { original cartridges. The following data were observed:

New cartridge	Existing cartridge
x_1 = 288 pages	x_2 = 279 pages
s_1 = 16:3 pages	s_2 = 15:91 pages

Based on the sample data and a signi cance level equal to 0.10, determine if the new system will result in a longer lasting product.

Answers.

1. The probability that the second card drawn is a heart is 1=4.
2. There are 3003 di erent committees that can be formed.
3. (a) $P(A \text{ and } B)$ = 0:22. (b) $P(A|B)$ = 0:5. (c) $P(A \text{ or } B)$ = 0:94. (d) $P(A^c \text{ and } B^c)$ = 0:06.
4. (a) Yes: all probabilities are between 0 and 1, and their sum is 1. (b) He can expect to sell 1.5 dishwashers during a day. (c) The standard deviation is 1.0149 dishwashers.
5. (a) The probability that exactly 5 of the 12 children live in homes with two married parents is 3.96%. (b) The probability that at most 2 of the 12 children live in homes with two married parents is 0.0374%. (c) The expected value is 8.16 children and the standard deviation is 1.616 children.
6. (a) P(1:1 < z < 0:72) = 0:1001.
7. (a) P(29 < X < 36) = 0:1557: (b) P(X < 40) = 0:5: (c) P(X > 32) = 0:9772:
8. (a) X is normally distributed with $_X$ = = 40; 000 and $_X$ = = p n = 1500= p

Formulas

Counting $P_{n,r} = \frac{n!}{(n-r)!}$ $C_{n,r} = \frac{n!}{r!(n-r)!}$

Random variable $\mu = E(X) = \sum_x x P(x)$ $\sigma = \sqrt{\sum_x (x-\mu)^2 P(x)} = \sqrt{\left[\sum_x x^2 P(x)\right] - \mu^2}$

Binomial variable $P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x}$ $\mu = np$ $\sigma = \sqrt{npq}$

Sample mean and standard deviation $\bar{x} = \frac{\sum x}{n}$ $s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

Z-scores: For parent population: $z = \frac{x-\mu}{\sigma}$ $x = \sigma z + \mu$

For sampling distribution: $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

Statistics for one sample mean: $E = z_c \frac{\sigma}{\sqrt{n}}$ $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ $n \geq \left(\frac{z_c \sigma}{E}\right)^2$

$E = t_c \frac{s}{\sqrt{n}}$ $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$ $n \geq \left(\frac{t_c s}{E}\right)^2$

Statistics for one sample proportion: $\hat{p} = \frac{r}{n}$ $\hat{q} = 1 - \hat{p}$

$E = z_c \sqrt{\frac{\hat{p}\hat{q}}{n}}$ $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$

Statistics for two sample means, dependent samples (or paired differences):

$\bar{d} = \frac{\sum d}{n}$; where $d = x_1 - x_2$ $s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$

$E = t \frac{s_d}{\sqrt{n}}$ $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$

Statistics for two sample means, independent samples:

$E = z \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ $z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

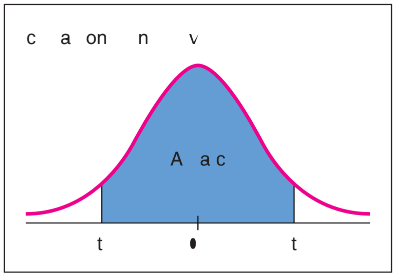
$E = t \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

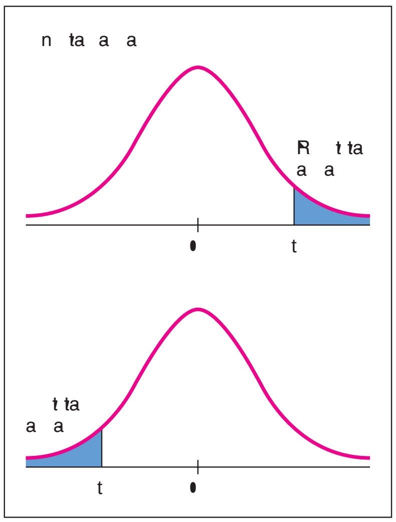
.0008	.0008	.0008	.0007	.0007	−3.1	.0010	.0009	.0009	.0009	.0008

Standard Normal Probabilities

.4982 .4983 .4984 .4984 .4985 .4985 .4986 .4986

2.9 .4981 .4982





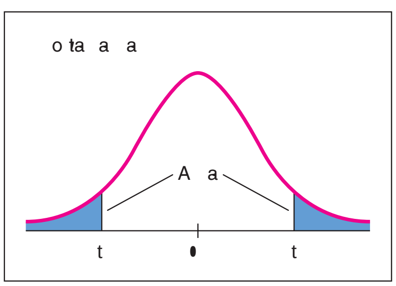


TABLE 6 Critical Values for Student's t Distribution

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ c	0.500	0.750	0. 00	0. 50	0.,00	0.,50	0., 0	0.,,0	0.,,,
1	1.000	2.414	3.07	4.165	6.314	12.706	31. 21	63.657	636.61,
2	0. 16	1.604	1. 6	2.2 2	2.,20	4.303	6.,65	,.,25	31.5,,
3	0.765	1.423	1.63	1.,24	2.353	3.1 2	4.541	5. 41	12.,24
4	0.741	1.344	1.533	1.77	2.132	2.776	3.747	4.604	.610
5	0.727	1.301	1.476	1.6,,	2.015	2.571	3.365	4.032	6. 6,
6	0.71	1.273	1.440	1.650	1.,43	2.447	3.143	3.707	5.,5,
7	0.711	1.254	1.415	1.617	1. ,5	2.365	2.,,	3.4,,	5.40
	0.706	1.240	1.3,7	1.5,2	1. 60	2.306	2. ,6	3.355	5.041
,	0.703	1.230	1.3 3	1.574	1. 33	2.262	2. 21	3.250	4.7 1
10	0.700	1.221	1.372	1.55,	1. 12	2.22	2.764	3.16,	4.5 7
11	0.6,7	1.214	1.363	1.54	1.7,6	2.201	2.71	3.106	4.437
12	0.6,5	1.20,	1.356	1.53	1.7 2	2.17,	2.6 1	3.055	4.31
13	0.6,4	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.6,2	1.200	1.345	1.523	1.761	2.145	2.624	2.,77	4.140
15	0.6,1	1.1,7	1.341	1.517	1.753	2.131	2.602	2.,47	4.073
16	0.6,0	1.1,4	1.337	1.512	1.746	2.120	2.5 3	2.,21	4.015
17	0.6 ,	1.1,1	1.333	1.50	1.740	2.110	2.567	2. ,	3.,65
1	0.6	1.1 ,	1.330	1.504	1.734	2.101	2.552	2. 7	3.,22
1,	0.6	1.1 7	1.32	1.500	1.72,	2.0,3	2.53,	2. 61	3. 3
20	0.6 7	1.1 5	1.325	1.4,7	1.725	2.0 6	2.52	2. 45	3. 50
21	0.6 6	1.1 3	1.323	1.4,4	1.721	2.0 0	2.51	2. 31	3. 1,
22	0.6 6	1.1 2	1.321	1.4,2	1.717	2.074	2.50	2. 1,	3.7,2
23	0.6 5	1.1 0	1.31,	1.4 ,	1.714	2.06,	2.500	2. 07	3.76
24	0.6 5	1.17,	1.31	1.4 7	1.711	2.064	2.4,2	2.7,7	3.745
25	0.6 4	1.1,	1.316	1.4 5	1.70	2.060	2.4 5	2.7 7	3.725
26	0.6 4	1.177	1.315	1.4 3	1.706	2.056	2.47,	2.77,	3.707
27	0.6 4	1.176	1.314	1.4 2	1.703	2.052	2.473	2.771	3.6,0
2	0.6 3	1.175	1.313	1.4 0	1.701	2.04	2.467	2.763	3.674
2,	0.6 3	1.174	1.311	1.47,	1.6,,	2.045	2.462	2.756	3.65,
30	0.6 3	1.173	1.310	1.477	1.6,7	2.042	2.457	2.750	3.646
35	0.6 2	1.170	1.306	1.472	1.6,0	2.030	2.43	2.724	3.5,1
40	0.6 1	1.167	1.303	1.46	1.6 4	2.021	2.423	2.704	3.551
45	0.6 0	1.165	1.301	1.465	1.67,	2.014	2.412	2.6,0	3.520
50	0.67,	1.164	1.2,,	1.462	1.676	2.00,	2.403	2.67	3.4,6
60	0.67,	1.162	1.2,6	1.45	1.671	2.000	2.3,0	2.660	3.460
70	0.67	1.160	1.2,4	1.456	1.667	1.,,4	2.3 1	2.64	3.435
0	0.67	1.15,	1.2,2	1.453	1.664	1.,,0	2.374	2.63,	3.416
100	0.677	1.157	1.2,0	1.451	1.660	1., 4	2.364	2.626	3.3,0
500	0.675	1.152	1.2 3	1.442	1.64	1.,65	2.334	2.5 6	3.310
1000	0.675	1.151	1.2 2	1.441	1.646	1.,62	2.330	2.5 1	3.300
`	0.674	1.150	1.2 2	1.440	1.645	1.,60	2.326	2.576	3.2,1

For degrees of freedom d.f. not in the table, use the closest d.f. that is smaller.