9. (12 marks) A and B are 3×3 matrices and $\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, and $\det B = 3$. Find

a) $\det A^T (2B)^3 \det(B)$ b) $\det 5BA^{-1} - 2B\,adj(A)$

c) $\begin{vmatrix} 2a & 3g & 5g & 4a & d \\ 2b & 3h & 5h & 4b & e \\ 2c & 3i & 5i & 4c & f \end{vmatrix}$

10. (4 marks) Evaluate the determinant $\begin{vmatrix} 2 & 1 & 5 & 1 \\ 8 & 0 & 1 & 3 \\ 1 & 1 & 6 & 2 \\ 3 & 1 & 5 & 3 \end{vmatrix}$ by row reduction. **You must perform at least one row operation.**

11. (9 marks) Let $\vec{u} = \langle 2, 1, 3 \rangle$, $\vec{v} = \langle 1, 2, 4 \rangle$, $\vec{w} = \langle 3, 1, \quad \rangle$.

a) Find the orthogonal projection of the vector $\vec{w}$ on the vector $\vec{u} \times \vec{v}$, that is $Proj_{\vec{u} \times \vec{v}} \vec{w}$.

b) Find a unit vector perpendicular to $\vec{u}$ and $\vec{v}$ and $\vec{w}$.

c) Find the area of a triangle determined by $\vec{u}$ and $\vec{v}$ and $\vec{w}$.

12. (1+3+3 marks) Given the point $A(3, \quad, 4)$, the plane P: $x + 3y - 2z = 4$ and the line L1: $\begin{matrix} x = 3 + t \\ y = 6 - t \\ z = 1 + 2t \end{matrix}$.

a) Determine whether the line is parallel to the plane.
b) Find the point on the plane P which is closest to the point A.
c) Find the distance from the point A to the plane P.

13. (1+3 marks) Show that the planes are not parallel and find the parametric equations of the line of intersection of the planes $5 + 3 = 4$ and $2x - 9y - 8z = 5$.

14. (3 marks) Simplify $\vec{a} \times 5\vec{b} \times 2\vec{a} \times 3\vec{b} \times \vec{a} \times \vec{b}$.

15. (6 marks) a)Show that L1: $\begin{matrix} x = 3 + t \\ y = 6 - t \\ z = 1 + 2t \end{matrix}$ and L2: $\begin{matrix} x = 1 + 2u \\ y = 3 + u \\ z = 4 + 3u \end{matrix}$ are skew lines

b) Find the equation of the plane containing the point $A(3, \quad, 4)$ and the line L1.

16. (8 marks) Maximize $P = 5x_1 + 4x_2 + 7x_3 + x_4$ subject to

17. (7 marks) Minimize $C = 5x_1 + 3x_2 + 18x_3$ subject to $\begin{matrix} x_1 + x_2 + 4x_3 \geq 20 \\ 4x_1 + 3x_2 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{matrix}$

Answers

1. a) $x_1 \quad 2 \quad 3t \quad s,\ x_2 \quad 1 \quad t \quad 5s,\ x_3 \quad t,\ x_4 \quad s.$ b) $x_1 \quad 1,\ x_2 \quad 4,\ x_3 \quad 0,\ x_4 \quad 1.$

2. a) $A^{1} \quad \begin{matrix} 1 & \frac{1}{3} & \frac{5}{3} \\ 1 & 0 & 1 \\ 1 & \frac{2}{3} & \frac{4}{3} \end{matrix}$; b) $x \quad 1,\ y \quad 2,\ z \quad 1.$

3. $y \quad 2$

4. $4A^{2}$

5. a) -47; b) $X \quad \begin{matrix} 3 & 4 \\ 2 & 3 \end{matrix}$

6. $E_1 \quad \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$, $E_2 \quad \begin{matrix} 1 & 0 \\ 2 & 1 \end{matrix}$. *Other possible answers.*

7. 1) $k \quad 3$; 2) $k \quad 3$; 3) $k \quad 3$

8. False.

9. a) $\frac{1}{128}$ b) $\frac{81}{4}$; c) 40

10. -16

11. a) $\frac{51}{}$, $\frac{51}{}$, $\frac{17}{}$; b) $\frac{7}{\sqrt{94}}, \frac{6}{\sqrt{94}}, \frac{3}{\sqrt{94}}$; c) $\sqrt{94}$.

12. a) Yes, the line is parallel to the plane; b) ; c) $2\sqrt{14}$.

13. $x \quad 11 \quad 13t,\ y \quad 3 \quad 2t,\ z \quad t$.

14. 0

15. b) $29x \quad 3y \quad 13z \quad 118 \quad 0$

16. $P \quad 43,\ x_1 \quad 3,\ x_2 \quad 0,\ x_3 \quad 4,\ x_4 \quad 0,.$

17. $C \quad 60,\ x_1 \quad 0,\ x_2 \quad 20,\ x_3 \quad 0.$