DAWSON COLLEGE
Mathematics Department
Final Examination
Linear Algebra
201ïNYCï05 (Commerce)
Fall 2021

1. a) (5 marks) Use Gauss-Jordan elimination to find the general solution of the system.
   b) (1 mark) Find a particular solution in which $x_2 \quad 5$ and $x_3 \quad 0$.

$$\begin{array}{ccccc} x_1 & 2x_2 & 2x_3 & 4x_4 & 3 \\ 3x_1 & 6x_2 & 5x_3 & 14x_4 & 1 \\ 4x_1 & 8x_2 & 6x_3 & 20x_4 & 4 \end{array}$$

2. (6+4 marks) Given the system of linear equations $\begin{array}{ccccc} 2x & y & 3z & 3 \\ x & 4y & z & 6 \\ 2x & 4y & z & 11 \end{array}$.

   a) Solve the system using the inverse matrix. Use the **adjoint** matrix to find the inverse.

   b) Use to solve the system **<u>for ñyò only</u>**.

3. (4 marks) Let $A \begin{matrix} 1 & 3 \\ 2 & 1 \\ 0 & 2 \end{matrix}$, $B \begin{matrix} 2 & 1 \\ 4 & 2 \\ 1 & 3 \end{matrix}$, $C \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$ .Solve for X : $CX^{1} \quad I^{1} \quad XA^TB$.

4. (3 marks) If $A \begin{matrix} 4 & 2 \\ 1 & 3 \end{matrix}$ and $B \begin{matrix} 2 & 1 \\ 7 & 7 \end{matrix}$, find elementary matrices E_1 and E_2 such that $E_2E_1A \quad B$.

5. (3 marks) Simplify as much as possible $B \quad 2C^{1}B^{1} \quad D^TC^{1} \quad 4A^TD^{T} \quad I \quad BD^0 \quad 2A^{1}$.

6. (4 marks) Determine the values of a such that the system has
   1) a unique solution, 2) infinitely many solutions, 3) no solution :

$$\begin{array}{l} x \quad 2y \quad 4z \quad 3 \\ \quad y \quad 7az \quad 2 \\ x \quad 3y \quad a^2 \quad 2 \; z \quad a \quad 6 \end{array}$$

7. (3 marks) Let A be an invertible

16. (7 marks) Maximize $P \quad 2x_1 \quad x_2 \quad 4x_3$ subject to
$$\begin{matrix} 4\ {}_1 \quad {}_2 \quad {}_3 \quad 3 \\ {}^1 \quad {}^3 \quad 2 \\ 3\ {}_1 \quad 2\ {}_2 \quad {}_3 \quad 15 \\ {}_1,\ {}_2,\ {}_3 \quad 0 \end{matrix}$$

17. (7 marks) Minimize $C \quad 2x_1 \quad 9x_2$ subject to
$$\begin{matrix} 2x_1 \quad 5x_2 \quad 2 \\ x_1 \quad 3x_2 \quad 5 \\ 3x_1 \quad x_2 \quad 1 \\ x_1, x_2 \quad 0 \end{matrix}$$

Answers

1. a) $x_1 \quad 13 \quad 2t \quad 8s,\ x_2 \quad t,\ x_3 \quad 8 \quad 2s,\ x_4 \quad s.$ b) $x_1 \quad 9,\ x_2 \quad 5,\ x_3 \quad 0,\ x_4 \quad 4.$

2. a) $A^{\ 1} \quad \begin{matrix} \frac{8}{19} & \frac{13}{19} & \frac{11}{19} \\ \frac{1}{19} & \frac{4}{19} & \frac{1}{19} \\ \frac{12}{19} & \frac{10}{19} & \frac{7}{19} \end{matrix}$, $x \quad 1,\ y \quad 2,\ z \quad 1.$; b) $y \quad 2$

3. $X \quad A^T B \ ^1 C \quad \begin{matrix} \frac{3}{2} & \frac{3}{2} \\ \frac{11}{5} & \frac{23}{5} \end{matrix}$

4. $E_1 \quad \begin{matrix} \frac{1}{2} & 0 \\ 0 & 1 \end{matrix}$, $E_2 \quad \begin{matrix} 1 & 0 \\ 4 & 1 \end{matrix}$. *Other possible answers.*

5. I

6. 1) $a \quad 1,\ a \quad 6$; 2) $a \quad 1$; 3) $a \quad 6$

7. skew-symmetric

8. -57

9. a) 24; b) $\frac{729}{2}$; c) 32

10. a) ; b) $\frac{12}{7}, \ \frac{4}{7}, \frac{8}{7}$; c) 1.9.

11. a) $\frac{\sqrt{14}}{2}$; b) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$, $\sqrt{\ }$

12. a) $x \quad 4 \quad 3t,\ y \quad 5 \quad 5t,\ z \quad t$; b) $x \quad 2 \quad 3s,\ y \quad 1 \quad 5s,\ z \quad 3 \quad s$.

13. a) $\sqrt{5}$; b) 0, 1, 4

14. True

15. b) $\frac{}{\sqrt{}}$; c) $y \quad z \quad 4 \quad 0$

16. $P \quad 13, \ x_1 \quad 0, \ x_2 \quad 5, \ x_3 \quad 2.$

17. $C \quad 10,$

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