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Note:

- Write your ans
- Only Sharp EL
- Use the revers
- The exam has

Reserved for Marking

Q1	Q2	Q3	Q4
/12	/6	/4	/3

1. **[12 marks]** Find the limits, and if it doesn't
use $L'H\hat{o}pital$'s rule:

(a) $\displaystyle\lim_{x\to 0}\frac{\sqrt{16+5x}-4}{3x}$ $= \lim_{x \to 0} \frac{5x}{3x(\sqrt{1}}$

(b) $\displaystyle\lim_{x\to 5^+}(x^2-25)\cos\left(\frac{1}{x-5}\right)$

$-1 \leq \cos\left(\frac{1}{x-5}\right) \leq 1$

$-(x^2-25) \leq (x^2-25)\cos\left(\frac{1}{x-}\right.$

Since $\lim_{x\to 5^+} -(x^2-25) = \lim_{x\to 5}$

by Squeeze Theorem, $\lim_{x\to 5^+}$

(c) $\lim\limits_{x \to 4} \frac{x^2 - x - 12}{| x - 4 |}$

n.e

↓

$- \frac{x^2 - x - 12}{|x-4|} =$

$\frac{x^2 - x - 12}{|x-4|} =$

(d) $\lim\limits_{x \to \infty} \arctan\left(\frac{1}{x}\right.$

$= \arctan \lim\limits_{x \to \infty}$

2. **[6 marks]** Given the function f:

$$f(x) = \begin{cases} \frac{2x^2}{\ } \\ \\ \\ \end{cases}$$

Find the x-values where f is discon
each case. Refer to the definition of c

- $x = 1$ JUMP DISCONTINU

because $\lim_{x \to 1} f(x)$ d.n.e.

- $x = -1$ HOLE (REMOVABLE)

because $f(-1)$ d.n.e, $\lim_{x \to -1}$

- $x = 2$ INFINITE DISCONTI

because $f(2)$ d.n.e $\lim_{x \to 2} f$

5. **[12 marks]** Find the derivatives for the

(a) $y = \dfrac{\sqrt{3x-4}}{(5x+2)^5}$

$$y' = \frac{\frac{1}{2\sqrt{3x-4}} \cdot 3 \cdot (5x+2)^5}{((5x+2)^5}$$

(b) $y = \arctan(\pi 5^{3x}) sec(4x) + \cos^2 \left(\frac{1}{\csc} \right.$

$$y' = \frac{1}{1+(\pi 5^{3x})^2} \cdot \pi \cdot 5^{3x} \ln 5$$

$$\cdot \sec(4x) \tan(4x) \cdot 4 + 2 \cos$$

$$\cdot (-\csc x)(\cot x)$$

(c) y

Y

6. [4 ma

$Y = x^{\frac{1}{x}}$

$\ln y = \frac{1}{x}$

$Y' = \frac{1}{x}$

$Y'' = \frac{-\frac{1}{x}}{}$

7. [3 mar
(in deg

(a) Wh

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(b) Ho
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(c) Fin

$\lim_{t \to \infty}$ (

After

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8. **[5 marks]** Given $f(x) = 2\arctan x$ and $g(x) = \ln(1 + x^2) + \frac{\pi}{2} - \ln 2$, find whet they have any common tangent line at one and the same point, and if yes, find the e ation of the tangent(s).

$$f'(x) = g'(x)$$

$$\frac{2}{1+x^2} = \frac{2x}{1+x^2}$$

$$2x = 2$$

$$x = 1$$

$$f(1) = g(1) = \frac{\pi}{2}$$

Point of common tangent $\left(1, \frac{\pi}{2}\right)$

Slope of the tangent : $f'(1) = g'(1) = 1$

Equation of the tangent

$$y = ax + b$$

$$\frac{\pi}{2} = 1 + b$$

$$b = \frac{\pi}{2} - 1$$

$$\boxed{y = x + \frac{\pi}{2} - 1}$$

9. [5 marks] I

at the poin

$\frac{y + xy}{xy}$

Replace

$1 + y'|_{(}$

$y'|_{(1,1)}$

$y = a$

$1 = 1 \cdot 1$

$b = 0$

$\boxed{y = x}$

[illegible]

11. [3 marks]
give a cou

(a) Ratior

Fals

is n

(b) Polyno

True

and c

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(c) Any ir

Fals

$x = 0$

12. **[8 marks]** Find the

(a) $\lim\limits_{x\to 0} \left(\frac{1}{\ln(x+1)} \right.$

$= \lim\limits_{x \to 0} \frac{x - \ell}{x}$

$\overset{0/0}{\underset{L'H}{=}} \lim\limits_{x\to 0} \frac{\frac{1}{(x}}{\frac{x+}{(x}}$

(b) $\lim\limits_{x\to 0^+} e^{\frac{1}{x}} \ln x$

(c) $\lim_{x\to 0^+} (2^x - 1)^x$ $\quad 0^0$

$$= e^{\ln \lim_{x\to 0^+} (2^x-1)^x} = e^{\lim_{x\to 0^+} x \ln(2^x-1)} = e^L$$

$$L = \lim_{x\to 0^+} \frac{\ln(2^x-1)}{\frac{1}{x}} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to 0^+} \frac{\frac{2^x \ln 2}{2^x-1}}{-\frac{1}{x^2}}$$

$$= -\ln 2 \lim_{x\to 0^+} \frac{2^x x^2}{2^x - 1} \underset{L'H}{\overset{\frac{0}{0}}{=}} -\ln 2 \lim_{x\to 0^+} \frac{\cancel{2^x} \ln 2\, x^2 + \cancel{2^x} \cdot 2x}{\cancel{2^x} \ln 2} =$$

$$= -\ln 2 \cdot 0 = 0$$

$$\lim_{x\to 0^+} (2^x - 1)^x = e^0 = 1$$

13. **[15 marks]** Consider the function:

$$f(x) = \frac{(1+x)^4}{(1-x)^4} \text{ with } f'(x) = \frac{8(1+x)^3}{(1-x)^5} \text{ and } f''(x) = \frac{16(1+x)^2(x+4)}{(1-x)^6}.$$

(a) Find its x- and y- intercepts.

x-int. $y=0 \Rightarrow x=-1$ $(-1,0)$

y-int: $x=0 \Rightarrow y=1$ $(0,1)$

(b) Find its asymptotes, if any. Justify your answer using limits.

Vertical $x=1$ $\lim_{x \to 1} \frac{(1+x)^4}{(1-x)^4} = +\infty$

Horizontal $y=1$ $\lim_{x \to \pm\infty} \frac{(1+x)^4}{(1-x)^4} = 1$

(c)

(d)

$f''($

x	
f''	
f	

f con

f con

Point

$y=1$

14.

A

15. **[6 marks]** Find the antiderivatives:

(a) $\displaystyle\int 3x^2\left(\frac{4}{x^3}+\frac{5}{\sqrt{x}}+8x^7\right)dx$

$= \int \left(12x^{-1} + 15x^{3/2} + 24x^9\right) dx$

$= 12 \ln|x| + 6x^{5/2} + \frac{12}{5}x^{10} + C$

(b) $\displaystyle\int (\sin x \cos x)e^{\sin^2 x}dx \quad = \frac{1}{2}e^{\sin^2 x} + C$

16. **[4 marks]** Giver

$f'(\theta) = \int (s$

$f'(0) = 0$

$f(\theta) = \int (-\omega$

$f(0) = 0$

So $f(\theta) = -s$